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DIFFRACTION OF KELVIN WAVES IN A ROTATING SEMIBOUNDED BASIN CONTAINING A SEMI-INFINITE WALL*

V.I. PLIS

The Wiener-Hopf method is used to obtain the exact solution of the problem of Kelvin wave diffraction in a rotating semibounded basin containing a semi-infinite wall. The solution is analysed asymptotically and numerically. The nature of the waves propagated in the basin is discussed.

1. Formulation of the problem. In the basin $-\infty < x < +\infty$, $-\infty < y < a$, located in a flat earth, rotating counter-clockwise with angular velocity 2ω, let there be a semi-infinite wall $y = 0$, $-\infty < x < 0$ (Fig.1). The depth of the basin is constant and equal to h. The axis of rotation is perpendicular to the (x, y) plane and passes through the origin $(0, 0)$.

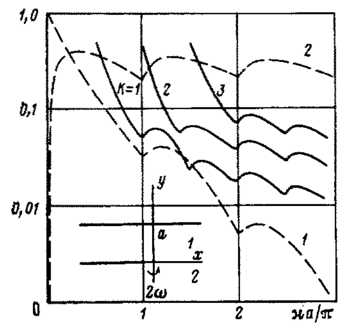

Fig.1

We consider the steady wave motions of a fluid surface in the basin, i.e., we assume that the rise $\xi(x, y, t)$ depends harmonically on time, $\xi(x, y) \exp(-i\sigma t)$. We will consider the case when $\sigma > 2\omega$. In the linear theory of long surface waves /1/, the function $\xi(x, y)$ is the solution of the Helmholtz equation

$$(\Delta + \varkappa^2)\xi(x, y) = 0, \quad \varkappa^2 = (\sigma^2 - 4\omega^2)/(gh)$$

where g is the acceleration due to gravity, and Δ is the two-dimensional Laplace operator.

Assume that a Kelvin wave of unit amplitude (1.1) propagates in the channel formed by the infinite and semi-infinite walls:

$$\xi_0(x, y) = \exp(i\eta\varkappa x - l\eta\varkappa y) \tag{1.1}$$

$$l = \frac{2\omega}{\sigma} < 1, \quad \eta = (1 - l^2)^{-1/2}$$

We shall study the wave motions in the basin which are excited when this wave is diffracted at the rib of the semi-infinite wall.

We divide the basin into two domains, see Fig.1. In domain 1 $(-\infty < x < +\infty,\ 0 < y < a)$ we write the total amplitude of the rise as $\xi_0 + \xi_1$, where ξ_0 are the incident, and ξ_1 the diffracted, waves. In domain 2 $(-\infty < x < +\infty,\ -\infty < y < 0)$ we denote the total amplitude of the rise by ξ_2. For unknown functions $\xi_j\ (j = 1, 2)$ we obtain the following problem: to find the solutions of the equations

$$(\Delta + \varkappa^2)\xi_j(x, y) = 0 \quad (j = 1, 2) \tag{1.2}$$

which satisfy the boundary conditions on the walls in contact with the fluid, and the conditions for continuity of the normal component of the velocity and rise on the continuation of the semi-infinite wall:

$$v_0(x, a-0) + v_1(x, a-0) = 0, \quad -\infty < x < +\infty \tag{1.3}$$

$$v_0(x, 0+0) + v_1(x, 0+0) = 0, \quad v_2(x, 0-0) = 0,$$
$$-\infty < x < 0$$

$$v_0(x, 0+0) + v_1(x, 0+0) = v_2(x, 0-0), \quad 0 < x < +\infty \tag{1.4}$$
$$\xi_0(x, 0+0) + \xi_1(x, 0+0) = \xi_2(x, 0-0), \quad 0 < x < +\infty$$

Here, $v_j(x, y)$ is the fluid velocity component, parallel to the y axis and connected with $\xi_j(x, y)$ by the relation

$$v_j(x, y) = -\frac{\sigma}{\varkappa^2 h}\left(l\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\xi_j(x, y) \tag{1.5}$$

Finally, the diffracted waves must satisfy the condition on the rib /2/

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$$\xi_j \sim r^{1/3}, \quad r \to 0, \quad r = \sqrt{x^2 + y^2} \tag{1.6}$$

and the following radiation condition: the solution at infinity must contain only divergent waves.

It can be shown that there is a unique solution of problem (1.1)–(1.6) in the class of bounded functions.

2. The system of paired integral equations and its solution. We shall solve problem (1.1)–(1.6) by the Wiener-Hopf method /3/. For this, we assume that the wave number $\varkappa$ has a small positive imaginary part, i.e., $\varkappa = \varkappa_0 + i\varepsilon$, and we let ε tend to zero in the final results. The introduction of the small imaginary term into $\varkappa$ corresponds to assuming energy dissipation in the fluid.

We will introduce the unknown functions $A(\alpha), B(\alpha), Z(\alpha), Z_1(\alpha), Z_2(\alpha)$ of the complex variable α by the relations

$$\xi_1(x, y) = \int_{-\infty}^{+\infty} \exp(i\alpha x)\,[A(\alpha)\sin\gamma(y-a) + B(\alpha)\sin\gamma y]\,d\alpha \tag{2.1}$$

$$\xi_2(x, y) = \int_{-\infty}^{+\infty} \exp(i\alpha x - i\gamma y)\,Z(\alpha)\,d\alpha$$

$$\xi_1(x, a) = \int_{-\infty}^{+\infty} \exp(i\alpha x)\,Z_1(\alpha)\,d\alpha, \quad \xi_2(x, 0) = \int_{-\infty}^{+\infty} \exp(i\alpha x)\,Z_2(\alpha)\,d\alpha$$

$$\left(A(\alpha) = -\frac{Z_2(\alpha)}{\sin\gamma a}, \quad B(\alpha) = \frac{Z_1(\alpha)}{\sin\gamma a}; \right.$$
$$\left. \gamma = (\varkappa^2 - \alpha^2)^{1/2}, \quad \operatorname{Im}\gamma > 0\right)$$

From the no-flow condition (1.3) on the wall $y = a$ we obtain

$$Z_1(\alpha) = \frac{\gamma Z_2(\alpha)}{\gamma\cos\gamma a + \alpha l\sin\gamma a} \tag{2.2}$$

We introduce the new unknown function $V(\alpha)$ of the complex variable α by

$$v_1(x, 0) = -\frac{\sigma}{\varkappa^2 h}\int_{-\infty}^{+\infty} \exp(i\alpha x)\,V(\alpha)\,d\alpha \tag{2.3}$$

On applying (1.5) to the integral forms for rises (2.1), we obtain in the light of (2.2), (2.3), the dependences of $Z(\alpha)$ and $Z_2(\alpha)$ on $V(\alpha)$

$$Z(\alpha) = \frac{V(\alpha)}{\gamma + i\alpha l} \tag{2.4}$$

$$Z_2(\alpha) = i\,\frac{\gamma\cos\gamma a + \alpha l\sin\gamma a}{\sin\gamma a\,(\gamma^2 + \alpha^2 l^2)}\,V(\alpha)$$

Substituting the integral forms of the rises in the second boundary condition (1.4) and using the no-flow condition on the semi-infinite wall, we arrive at the system of paired integral equations

$$\int_{-\infty}^{+\infty} \frac{\exp(i\alpha x)\,L(\alpha)}{\alpha^2 - \eta^2\varkappa^2}\,V(\alpha)\,d\alpha = i\,\frac{a}{\eta^2}\exp(i\eta\varkappa x), \quad x > 0 \tag{2.5}$$

$$\int_{-\infty}^{+\infty} \exp(i\alpha x)\,V(\alpha)\,d\alpha = 0, \quad x < 0$$

$$L(\alpha) = \frac{\gamma a}{\sin\gamma a}\exp(-i\gamma a)$$

To solve this system, we factorize the kernel $L(\alpha)$, i.e., we write it as $L(\alpha) = L_+(\alpha)\,L_-(\alpha)$, where the function $L_+(\alpha)$ is analytic and has no zeros in the upper half-plane of α, while $L_-(\alpha)$ has the same properties in the lower half-plane of α. The functions $\sin\gamma a/(\gamma a)$ and $\exp(-i\gamma a)$ have been factorized in the literature, see e.g., /3, 4/, so that we shall just quote the final result for the kernel $L(\alpha)$:

$$\frac{1}{L_+(\alpha)} = \sqrt{\frac{\sin\varkappa a}{\varkappa a}}\exp\left[\frac{\alpha\gamma}{\pi}\ln\left(\frac{\alpha + i\gamma}{\varkappa}\right) + \frac{i\alpha a}{\pi}\times\right. \tag{2.6}$$
$$\left.\left(1 - C + \ln\frac{2\pi}{\varkappa a} + i\frac{\pi}{2}\right)\right]\prod_{n=1}^{\infty}\left(1 - \frac{\alpha}{\alpha_n}\right)\exp\frac{i\alpha a}{\pi n}, \quad \alpha_n = -\sqrt{\varkappa^2 - \left(\frac{\pi n}{a}\right)^2}$$

where $C = 0{,}57721\ldots$ is Euler's constant.

We will seek the solution of system (2.5) in the form

$$V(\alpha) = Q/L_-(\alpha) \tag{2.7}$$

where Q is an unknown constant. With this choice of $V(\alpha)$ the second equation of (2.5) is satisfied identically. To find the constant Q, we substitute (2.7) into the second equation of (2.5) and, after evaluating the resiude at the pole $\alpha = \eta\varkappa$, we find

$$Q = \frac{\varkappa a}{\pi\eta L_+(\eta\varkappa)} \tag{2.8}$$

Our solution (2.7), (2.8) satisfies rib condition (1.6), which, by the theorem on the relation between the asymptotic forms of a function and its Fourier transform /5/, takes the form for the function $V(\alpha): V(\alpha) \sim \alpha^{-1/3}$ as $\alpha \to \infty$. Knowing the explicit expression for $V(\alpha)$, we can obtain expressions for the rises of the fluid surface in the basin.

3. Expressions for the rises. We start by studying the rise in the channel $-\infty < x < 0$, $0 < y < a$. Starting from (2.1), the following integral form can be obtained for the rise in the channel:

$$\xi_1(x, y) = i\eta^2 \int_{-\infty}^{+\infty} \frac{\exp(i\alpha x)\,[\gamma\cos\gamma(y-a) - \alpha l \sin\gamma(y-a)]}{\sin\gamma a\,(\alpha^2 - \eta^2\varkappa^2)} V(\alpha)\,d\alpha \tag{3.1}$$

To evaluate the integral in (3.1), it suffices to find the residues of the integrand at the simple poles $-\eta\varkappa$, α_k $(k = 1, 2, \ldots)$. We obtain

$$\xi_1(x, y) = -\frac{\pi l \eta^2}{\operatorname{sh}(l\eta\varkappa a)} V(-\eta\varkappa)\exp[i\eta\varkappa x + l\eta\varkappa(y-a)] + \tag{3.2}$$

$$2\pi i \sum_{k=1}^{\infty} \frac{\gamma_k}{\alpha_k a} \frac{V(\alpha_k)}{\delta_k} \sin(\gamma_k y + \varphi_k)\exp(i\alpha_k x)$$

$$\sin\varphi_k = \frac{\gamma_k}{\delta_k}, \quad \cos\varphi_k = \frac{\alpha_k l}{\delta_k}, \quad \delta_k = \sqrt{\gamma_k^2 + \alpha_k^2 l^2}, \quad \gamma_k = \frac{\pi k}{a}$$

The first term in (3.2) describes the reflected Kelvin wave (KW), travelling in the channel, while the infinite sum refers to progressive and damped waves. Progressive waves correspond to real α_k and exponentially damped waves to imaginary α_k. Given the dimensionless channel width $\varkappa a$, the number of progressive waves is equal to the integral part of $\varkappa a/\pi$.

Let us turn to domain 2. The rises in it are described by the integral relation

$$\xi_2(x, y) = -\int_{-\infty}^{+\infty} \frac{\exp(i\alpha x - i\gamma y)}{\gamma + i\alpha l} V(\alpha)\,d\alpha \tag{3.3}$$

The integrand in (3.3) has branching points $\pm\varkappa$ and a simple pole $-\eta\varkappa$. For $x < 0$ we can use the residue theorem of /6/ and write the rise $\xi_2(x, y)$ as

$$\xi_2(x, y) = -2\pi l \eta^2 V(-\eta\varkappa)\exp(i\eta\varkappa x + l\eta\varkappa y) + \int_S \frac{\exp(i\alpha x - i\gamma y)}{\gamma + i\alpha l} V(\alpha)\,d\alpha \tag{3.4}$$

The first term in (3.4) describes the KW, travelling in domain *2* in the negative x direction along the wall $y = 0$, $x < 0$, while the second term (the integral along the sides of the cut S to the branching points) describes the complex wave motion and can be computer-evaluated by numerical integration for any point of the domain.

On estimating integral (3.3) for the rise at a great distance compared with the wavelength from the channel input by the saddle-point method /7/, we obtain the following for the surface rise remote from the rib of the semi-infinite wall:

$$\xi_2(r, \theta) \sim \sqrt{\frac{2\pi}{\varkappa r}}\,\Psi(\theta)\exp\left[i\left(\varkappa r - \frac{\pi}{4}\right)\right], \quad \varkappa r \gg 1 \tag{3.5}$$

$$\Psi(\theta) = \frac{\cos\theta V(\varkappa\sin\theta)}{\cos\theta + il\sin\theta}, \quad x = r\sin\theta, \quad y = -r\cos\theta$$

The same estimate can be made for the rise in domain *1* outside the channel. It is clear from (3.5) that, remote from the rib of the semi-infinite wall, the rise is a cylindrical wave with an angular distribution of the amplitude $|\Psi(\theta)|$. It must be said that, for $x < 0$, a term describing the KW has to be added to the cylindrical waves in (3.5).

Let us now describe the wave picture as a whole. The KW travelling in the channel reaches the rib of the semi-infinite wall, swings round at it, and departs to infinity with reduced amplitude on the other side of the wall. Apart from this wave, cylindrical waves travel outside the channel. In the channel itself, a system of natural wave is excited, namely, the reflected KW, a finite number of progressive waves, and an infinite number of waves which are exponentially damped on moving deeper into the channel from its open end.

The problem of the diffraction of KW travelling along the infinite wall in the basin shown in Fig.1 was solved in /8/ by the Wiener-Hopf method in John's interpretation. It must be said that, in /8/, as also in /9/ on a similar topic by the same author, it was claimed that the amplitude of the n-th progressive wave in the channel tends to infinity as $\alpha_n \to 0$, i.e., as $\varkappa a \to \pi n$, and that the expressions for the rises are valid when $\varkappa a$ is not too close to πn, because, in expressions (3.2) for the rises of the progressive waves, the wave parameters α_n occur in the denominator. However, in the numerator we always have the factor $(\sin \varkappa a)^{1/2}$, which tends to zero as $\varkappa a \to \pi n$ at the same rate as α_n. Thus the amplitude of any progressive wave in the channel is always finite, whole solution (2.7), (2.8), (3.2) is valid for any $\varkappa a$.

Interpretation of results of a numerical analysis. The amplitudes of the waves arising in the basin were studied numerically. The infinite product in (2.6) was replaced by a finite product with N factors. It was shown in /10/ that the relative error of this reduction does not exceed $|\alpha^2| a^2/(\pi^2 N)$, so that, for values of $\alpha a/\pi \sim 1$, the error is under 1% when $N > 100$.

In Fig.1 the broken curves *1* and *2* are respectively the amplitude of reflected KW and KW travelling in domain *2*, plotted against the channel width $\varkappa a$ It can be seen that, for small $\varkappa a$ the KW, on reaching the open end of the channel, is almost completely reflected from it. As $\varkappa a$ increases, the reflected KW amplitude falls sharply, while the KW amplitude in the "shadow" domain tends to its limit, equal to $l\eta/(1+\eta)$ /11/. The explanation is that, as $\varkappa a \to +\infty$, our problem transforms into the KW diffraction problem at a semi-infinite wall in an unbounded rotating basin.

The continuous curves plot the amplitudes of several first progressive waves against the channel dimensionless width $\varkappa a$. As the width increases, with $\varkappa a = \pi n$ $(n = 1, 2, \ldots)$, progressive waves appear in the channel. The values of the dimensionless width at which excitation of a new travelling wave occurs are called threshold values. At these threshold values there are typical breaks in the amplitude curves, linked with the redistribution of energy between the waves close to the threshold at which a new wave appears. The effect of reconstruction of waves motions on the birth of a new travelling wave is familiar in optics /12/, electrodynamics /13/, and nuclear physics /14/, where it is known as a threshold effect. The threshold nature of diffraction of long surface waves in a rotating basin was discussed in detail in /10, 16-18/.

Our present results can be used in geophysical calculations when studying the motion of tidal waves, as e.g., in /15/.

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REFERENCES

1. SRETENSKII L.N., The theory of wave motions of a fluid (Teoriya volnovykh dvizhenii zhidkosti), ONTI, Moscow-Leningrad, 1936.
2. PLIS V.I., On the "rib condition" in the linear theory of long surface waves", PMM 43, 3, 1979.
3. NOBLE B., Methods based on the Wiener-Hopf technique, Pergamon, 1959.
4. MITTRA R. and LEE S., Analytic methods of waveguide theory /Russian translation/, Mir, Moscow, 1974.
5. RIEKSTIN'SH E.YA., Asymptotic expansions of integrals (Asimptoticheskie razlozheniya integralov), Zinatne, Riga, 1974.
6. LAVRENT'YEV M.A. and SHABAT B.V., Methods of the theory of functions of a complex variable (Metody teorii funktsii kompleksnogo peremennogo), Nauka, Moscow, 1973.
7. FEDORYUK M.V., The saddle-point method (Metod perevala), Nauka, Moscow, 1977.
8. KAPOULITSAS G.M., Diffraction of Kelvin waves from a rotating channel with an infinite and a semi-infinite barrier, J. Phys. Ser. A. 19, 5, 1979.
9. KAPOULITSAS G.M., Scattering of long waves in a rotating bifurcated channel, Intern. J. Theor. Phys., 19, 10, 1980.
10. PLISS V.I., Diffraction of Kelvin waves in a channel with a semi-infinite wall. PMM 47, 6, 1983.
11. GABOV S.A., Diffraction of Kelvin wave at a semi-infinite wall in an unbounded basin, Vestn. MGU, mekh., 5, 1973.
12. WOOD R.W., Diffraction gratings with a controlled groove form and an abnormal distribution of intensity, Philos. Mag. Ser. 6, 23, 133, 1912.
13. BOLOTOVSKII B.M. and LEBEDEV A.N., On threshold effects in classical electrodynamics, Zh. eksperim. i teor. fiziki, 53, 4, 1967.
14. BAZ' A.I., ZEL'DOVICH YA.B. and PERELOMOV A.M., Scattering, reaction and decay in non-relativistic quantum mechanics (Rassenie, raktsii i raspady v nerelyativitskoi kvantovoi mekhanike), Nauka, Moscow, 1966.
15. PACKHAM B.A., Reflection of Kelvin waves at the open end of a rotating semi-infinite channel, J. Fluid Mech., 39, pt.2, 1969.
16. GABOV S.A., RUBAN P.I. and SEKERZH-ZENKOVICH S.YA., Diffraction of Kelvin waves at a semi-infinite wall in a semibounded basin, Zh. vych. Mat. i mat. Fiz., 15, 6, 1975.

17. PLIS A.I. and PLIS V.I., Diffraction of Kelvin waves at the open end of a plane-parallel channel, PMM 44, 1, 1980.
18. PLIS V.I., Propagation of Kelvin waves from a channel into a semibounded basin, PMM 45, 6, 1981.

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INFLUENCE OF DISSIPATION ON THE PROPAGATION OF A SPHERICAL EXPLOSION SHOCK WAVE*

V.N. LIKHACHEV

The problem of the propagation of an explosion shock wave in a weakly compressible viscous medium at low Reynolds numbers is solved by the method of asymptotic expansions. The influence of non-linear terms in the principal approximation is studied, and the law of wave amplitude damping and its profile are found.

1. Formulation of the problem. The system of equations that describes the spherically symmetric motion of a compressible viscous fluid is /1/

$$\frac{\partial \bar u}{\partial \bar t} + \bar u \frac{\partial \bar u}{\partial \bar x} = -\frac{1}{\bar\rho}\frac{\partial \bar p}{\partial \bar x} + \frac{1}{\bar\rho}\left(\zeta + \frac{4}{3}\mu\right)\left(\frac{\partial^2 \bar u}{\partial \bar x^2} + \frac{2}{\bar x}\frac{\partial \bar u}{\partial \bar x} - \frac{2\bar u}{\bar x^2}\right) \tag{1.1}$$

$$\frac{\partial \bar\rho}{\partial \bar t} + \bar u \frac{\partial \bar\rho}{\partial \bar x} + \bar\rho\left(\frac{\partial \bar u}{\partial \bar x} + \frac{2\bar u}{\bar x}\right) = 0$$

$$\frac{\partial \bar s}{\partial \bar t} + \bar u \frac{\partial \bar s}{\partial \bar x} = \frac{1}{\bar\rho \bar T}\left\{\left(\zeta - \frac{2}{3}\mu\right)\left(\frac{\partial \bar u}{\partial \bar x} + \frac{2\bar u}{\bar x}\right)^2 + 2\mu\left[\left(\frac{\partial \bar u}{\partial \bar x}\right)^2 + \frac{2\bar u^2}{\bar x^2}\right] + \frac{\bar\varkappa}{\bar x^2}\frac{\partial}{\partial \bar x}\left(\bar x^2 \frac{\partial \bar T}{\partial \bar x}\right)\right\}$$

where the bar refers to dimensional quantities, $\bar s, \bar T$ are the entropy per unit mass and the temperature, $\zeta, \mu, \bar\varkappa$ are the coefficients of shift, spatial viscosity, and thermal conductivity. Knowing the internal energy as a function of $\bar\rho$ and $\bar s$ we can find the dependences $\bar p(\bar\rho, \bar s)$ and $\bar T(\bar\rho, \bar s)$. These relations close system (1.1).

The action of the explosion products on a fluid is modelled by a piston, moving according to the law $\bar x = \bar\varphi(\bar t)$, where $\bar\varphi(0) = \bar x_0$, $\bar\varphi'(0) = \bar U_0$ ($\bar U_0$ is the shock initial velocity).

We will introduce the dimensionless variables

$$\sigma = \frac{\bar p}{\bar\rho_0 \bar U_0^2},\quad \rho = \frac{\bar\rho}{\bar\rho_0},\quad u = \frac{\bar u}{\bar U_0},\quad x = \frac{\bar x}{\bar x_0},\quad t = \frac{\bar t \bar U_0}{\bar x_0},\quad T = \frac{\bar T}{\bar T_0},\quad s = \frac{\bar s \bar T_0}{\bar U_0^2}$$

where $\bar\rho_0$ and $\bar T_0$ are the density and temperature of the undisturbed medium.

The medium is assumed to be weakly compressible. We will introduce the small parameter $\varepsilon = (\partial\sigma/\partial\rho)_s^{-1}$ in the undisturbed medium, and solve the problem in the range of parameters ensuring small density disturbances: $\rho = 1 + \varepsilon p$. For inviscid flow, p is the same in the principal approximation as the dimensionless pressure /2/.

We shall seek the principal term of the expansion of the solution with respect to the small parameter ε. Neglecting terms in (1.1) that are obviously taken into account in later approximations, we obtain the system

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} - \left(\frac{\partial\sigma}{\partial s}\right)_\rho \frac{\partial s}{\partial x} + \alpha^0\left(\frac{\partial^2 u}{\partial x^2} + \frac{2}{x}\frac{\partial u}{\partial x} - \frac{2u}{x^2}\right) \tag{1.2}$$

$$\varepsilon\frac{\partial p}{\partial t} + \varepsilon u\frac{\partial p}{\partial x} + \frac{\partial u}{\partial x} + \frac{2u}{x} = 0$$

$$\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} = \frac{1}{T}\left\{\alpha_1^0\left(\frac{\partial u}{\partial x} + \frac{2u}{x}\right)^2 + \alpha_2^0\left[\left(\frac{\partial u}{\partial x}\right)^2 + \frac{2u^2}{x^2}\right] + \varkappa\frac{1}{x^2}\frac{\partial}{\partial x}\left(x^2\frac{\partial T}{\partial x}\right)\right\}$$

$$\alpha_1^0 = \frac{3\zeta - 2\mu}{3\bar\rho_0 \bar U_0 \bar x_0},\quad \alpha_2^0 = \frac{2\mu}{\bar\rho_0 \bar U_0 \bar x_0},\quad \alpha^0 = \alpha_1^0 + \alpha_2^0,\quad \varkappa = \frac{\bar\varkappa \bar T_0}{\bar\rho_0 \bar U_0^3}$$

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